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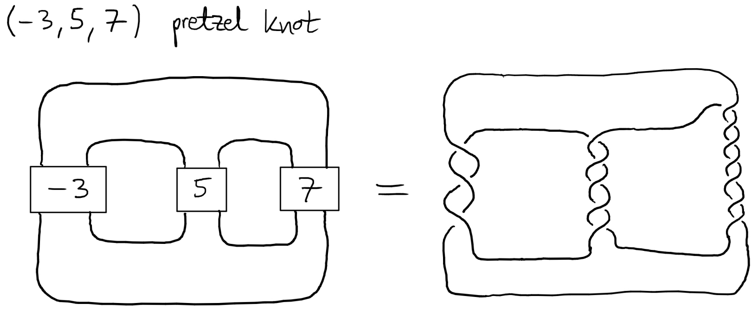
Low-dimensional topology, surfaces in 4-manifolds, knot concordance

Current Research Interests

Topology in four dimensions is particularly interesting because the setting is complicated enough for many intricate phenomena to arise, but in contrast to higher dimensions there is not enough room to untangle and simplify these. Moreover, from dim$= 4$ onward the difference between the topological and smooth category emerges; this is demonstrated by the observation that many constructions work topologically but fail to hold smoothly.

To illustrate this, let's start with a knot $K \subset \mathbb{S}^3$ and the question whether K is the boundary of a "nicely" embedded disk in the 4-ball. Freedman has a theorem stating that Alexander polynomial $\Delta_K(t) \doteq 1$ is enough to conclude that the knot in question is **topologically** slice[1]. For example, take a peek at the $(-3, 5, 7)$ pretzel knot which has a trivial Alexander polynomial.



Even though Freedman's result tells us that there is a topological slice disk for this particular knot, its Rasmussen s-invariant[2] is nonzero and so it does not have a **smooth** slice disk in $\mathbb{D}^4$. Now, such a creature (a topologically but not smoothly slice knot) gives rise to an exotic smooth structure on $\mathbb{R}^4$, and there are still many open questions regarding this relationship.

After completing my master's degree at the university of Bonn, I started working on my PhD in October 2018. Right now I like to manipulate surfaces in 4-manifolds with Finger and Whitney moves, think of slice disks which have to go deep into a 4-manifold and would like to do something with group trisections!

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[1] *Topologically slice* requires the embedding $\mathbb{D}^2 \hookrightarrow \mathbb{D}^4$ of the slice disk with $\partial\mathbb{D}^2 = K \subset \mathbb{S}^3$ to be locally flat, i.e. it can be thickened to $\mathbb{D}^2 \times \mathbb{D}^2 \hookrightarrow \mathbb{D}^4$ and this should restrict to a tubular neighborhood of the knot in $\mathbb{S}^3$.

[2] The *s-invariant* can be defined combinatorially using *Khovanov Homology*: That is a homology theory for knots which is motivated from a categorification of the Jones polynomial.