Casson – Whitney unknotting numbers

Unknotting knotted 2-spheres in S^4 with Finger & Whitney moves

with Jason Joseph, Michael Klug & Hannah Schwartz

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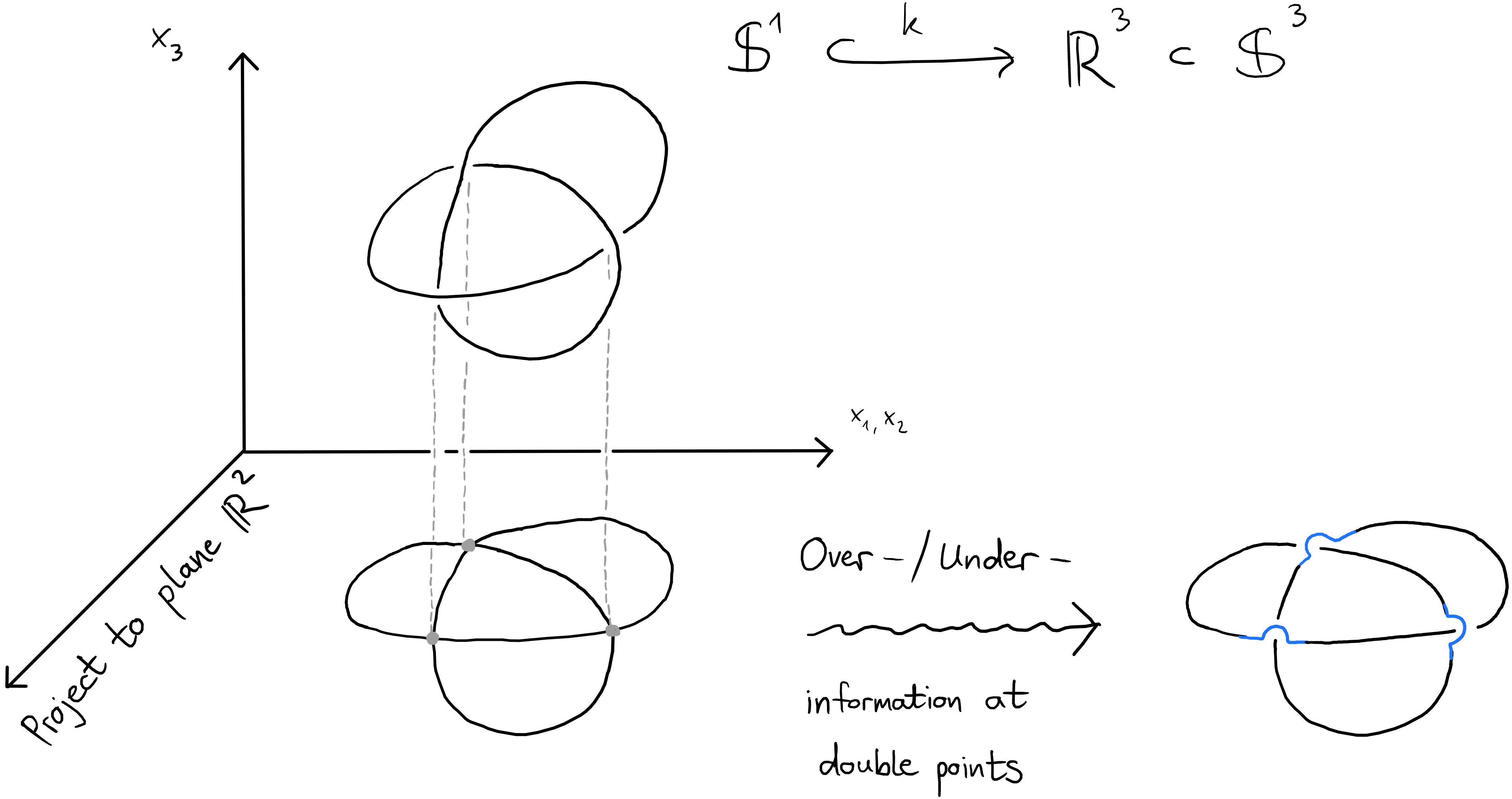
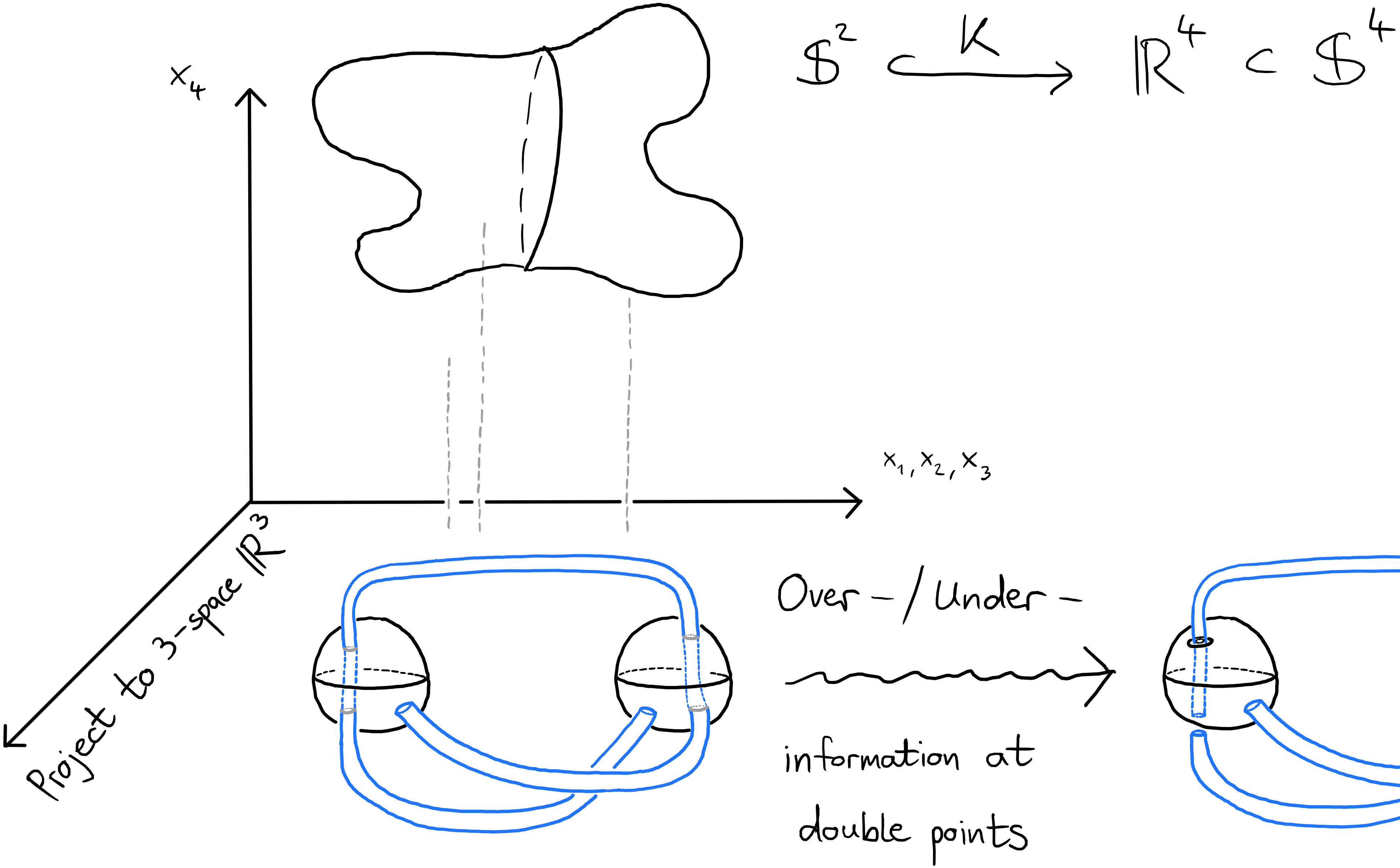
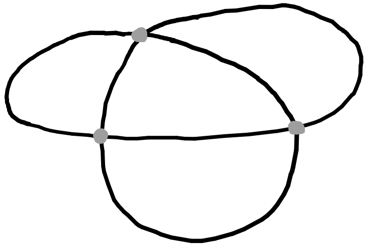
BIGS Lightning talk (7 min)


$\mathbb{S}^1 \xhookrightarrow{k} \mathbb{R}^3 \subset \mathbb{S}^3$
x_3
x_1, x_2
Project to plane $\mathbb{R}^2$
Over-/Under-
information at
double points

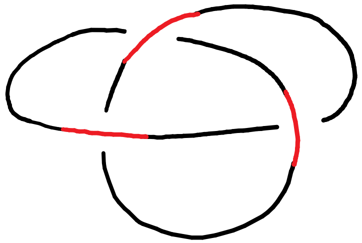


$\mathbb{S}^2 \overset{K}{\hookrightarrow} \mathbb{R}^4 \subset \mathbb{S}^4$
x_4
x_1, x_2, x_3
Project to 3-space $\mathbb{R}^3$
Over-/Under-
information at
double points

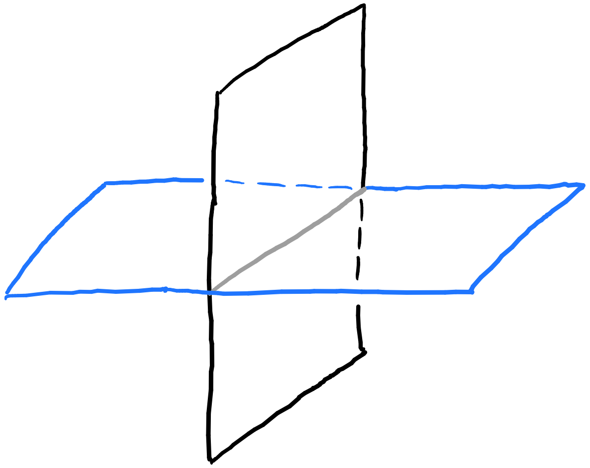


Over-/Under-

information at double points

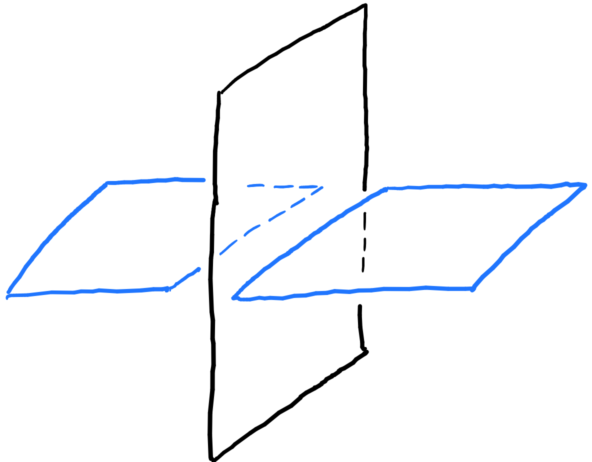


Broken surface diagrams



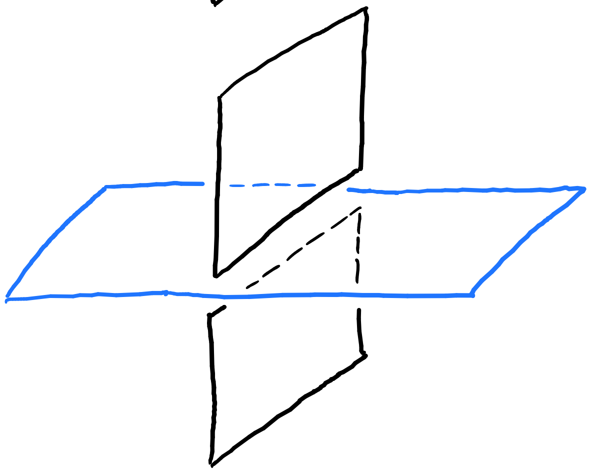
black sheet is "higher up"

$x_4 \gg x_4$

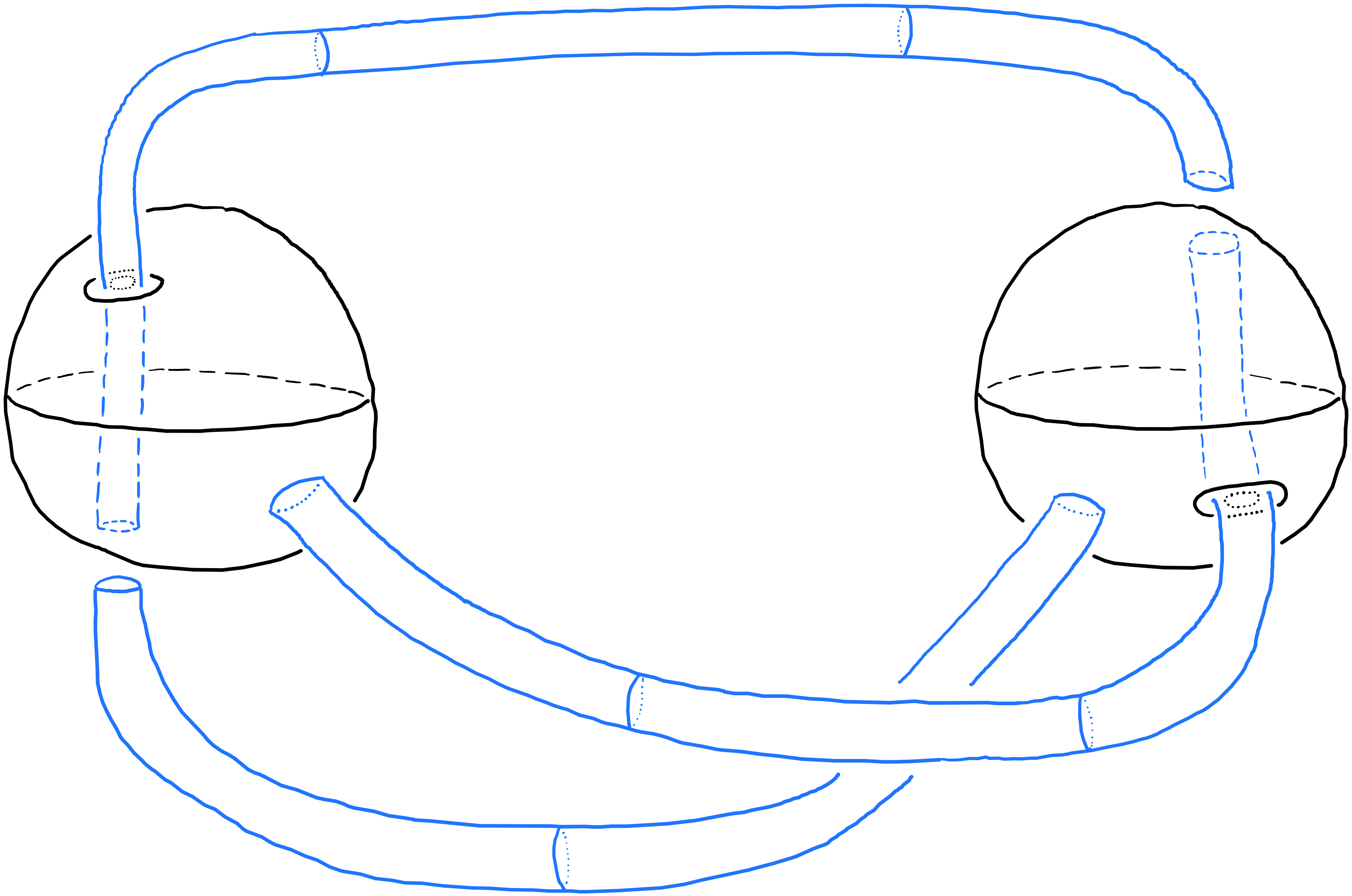


blue sheet is "higher up"

$x_4 \gg x_4$



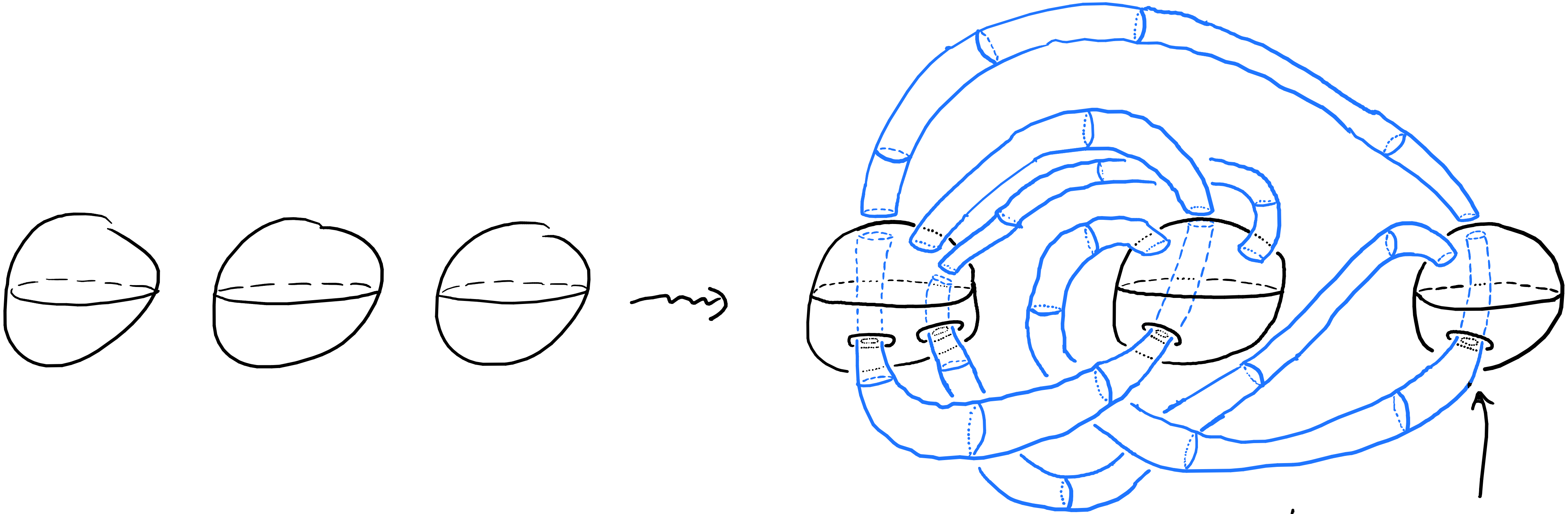
Ribbon 2-knots in 4-space



Ribbon 2-knots in 4-space

Start with an unlink of 2-spheres in S^4

Attach fusion tubes



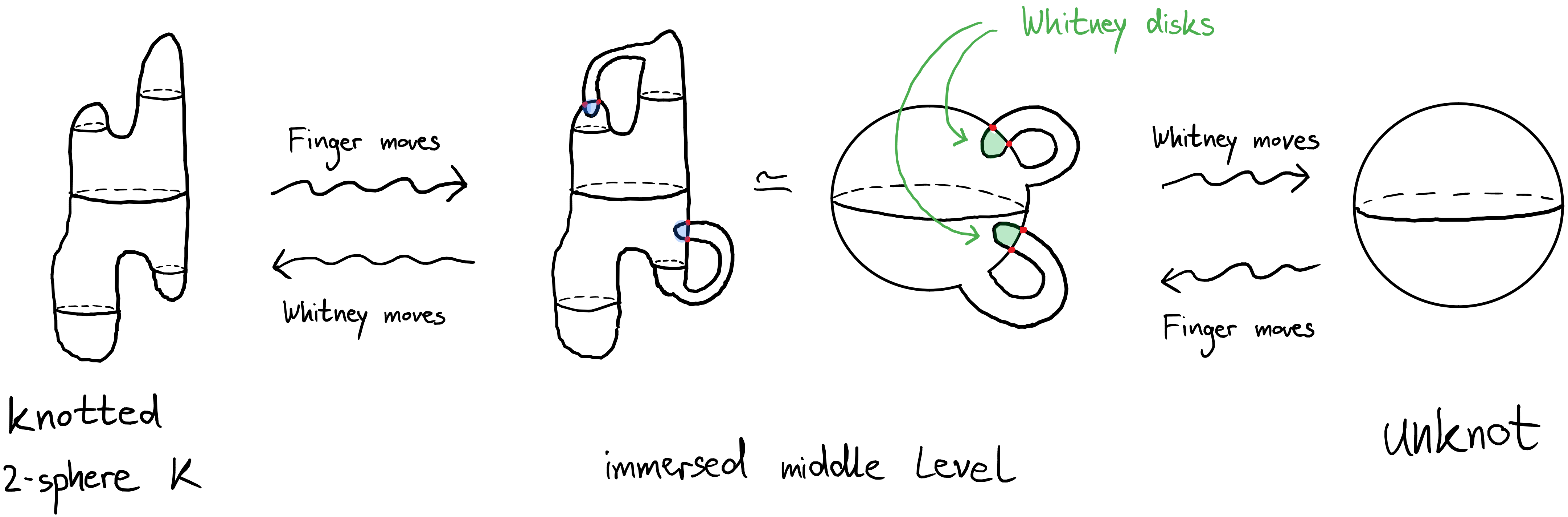
the blue tubes "link" with the black spheres

We [Joseph - Klug - R. - Schwartz] define the Casson-Whitney number

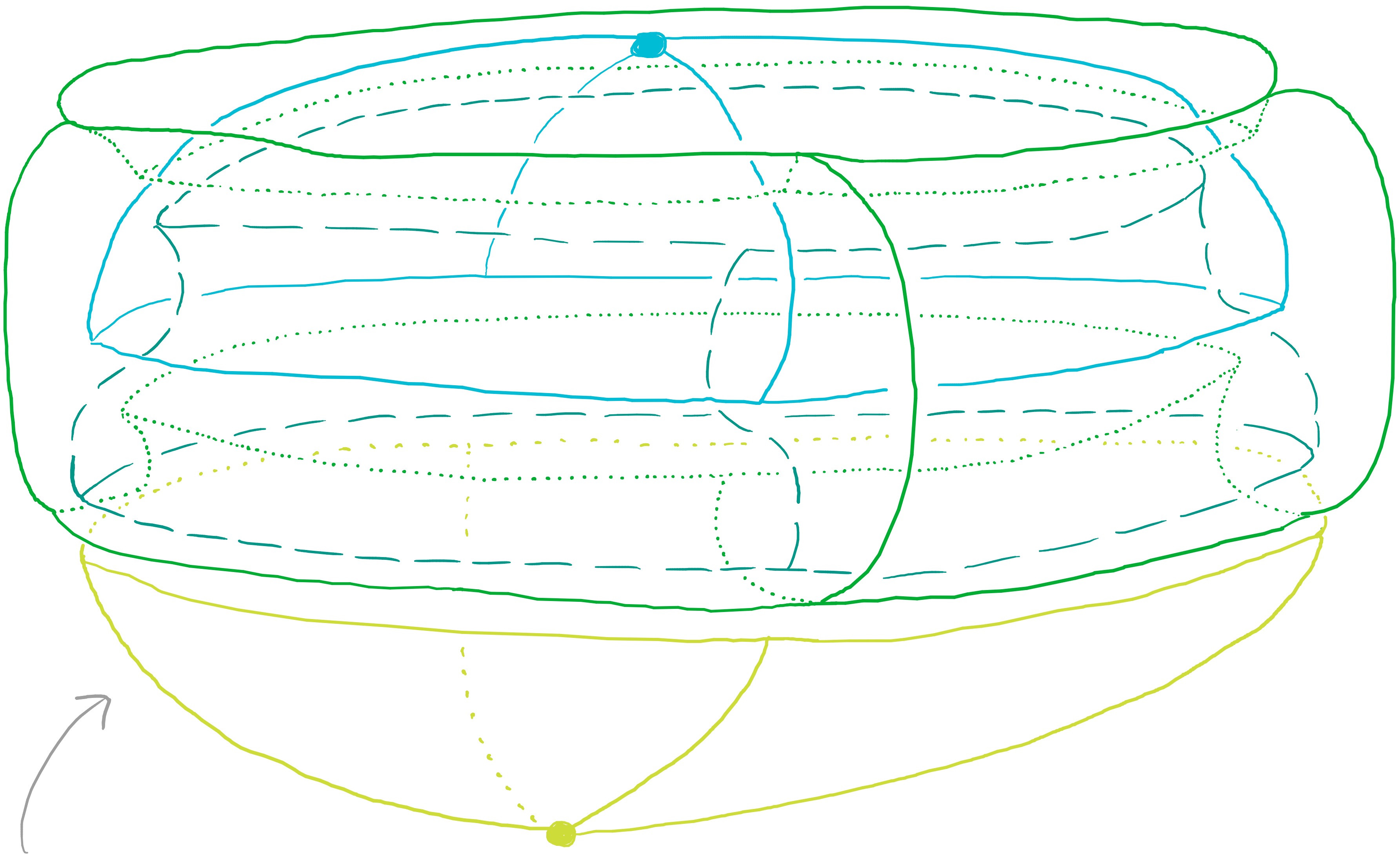
$$u_{CW}(K) \quad \text{of} \quad K : S^2 \hookrightarrow S^4$$

as the minimal number of Finger moves in a regular homotopy $K \rightsquigarrow$ unknot

Schematic of a regular homotopy:



Thanks!



broken surface diagram of a spun trefoil