Eight faces of the Poincaré Homology Sphere

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Website: http://ben300694.github.io
Kirby & Scharlemann’s paper from 1979

Poincaré’s original construction from 1904

Figure 1: Genus 2 Heegaard diagram from [Poi04], more on the history at the Manifold Atlas
Here Rolfsen [Rol03] demonstrates that $(-1)$-Dehn surgery on the trefoil describes the same 3-manifold as Poincaré’s Heegaard diagram.
Quaternions

3blue1bown’s interactive video series on visualizing quaternion multiplication

https://eater.net/quaternions
Filling a 3-sphere with 120 dodecahedrons

- Visualization of the 120-cell:
  https://www.youtube.com/watch?v=MFXRRW9goTs

**Figure 2:** 120-cell, image by Robert Webb’s Stella software
Figure 3: Leslie Valiant (Nevanlinna Prize), Michael Hartley Freedman, Gerd Faltings, Simon Donaldson (Fields Medalists), at the ICM 1986 in Berkeley. [Link]
Further reading

- Connections between Algebraic curve singularities and knot theory: [Poincaré homology sphere and exotic spheres from links of hypersurface singularities], [Knot Theory and Problems on Affine Space]
- Group trisections: [AGK18] with a purely group-theoretic formulation of the smooth 4-dimensional Poincaré conjecture
- Kirby calculus (graphical calculus for manipulating handle decompositions of 4-manifolds): [GS99]
- Rolfsen [Rol03]: The “Old Testament” of knot theory and low-dimensional topology


